ANALYSIS OF A DEEP UNSUPPORTED CIRCULAR UNDERGROUND OPENING WITH Z_SOIL.PC

1 INTRODUCTION

The elastoplastic solution for a deep underground circular opening in an initial isotropic stress state is discussed in this short paper. The numerical solution obtained with Z_Soil is compared with the analytical solution (see Hoek & Brown 1980 for the elastic solution and Panet 1995 for the elasto-plastic solution).

2 PROBLEM STATEMENT

Figure 1 shows the studied circular underground opening in flysch sandstones, at a depth of 800 m. An internal pressure p equal to the isotropic initial stress is initially applied at the tunnel's radius, and then decreased until reaching p = 0. An elastic perfectly-plastic Mohr-Coulomb type criterion is used. No coupling with water pressure is considered.

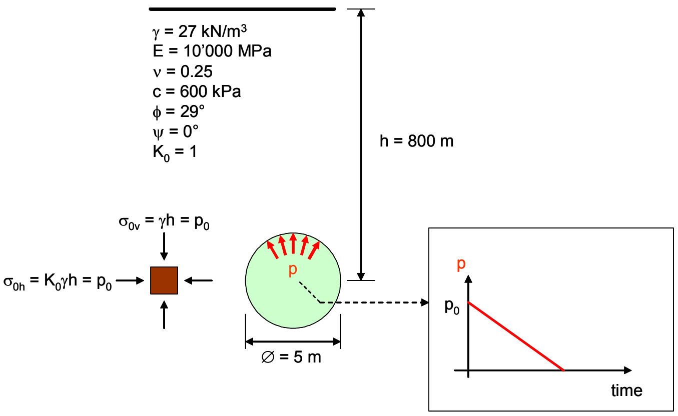

Figure 1. Problem statement.

2.1 *FE mesh*

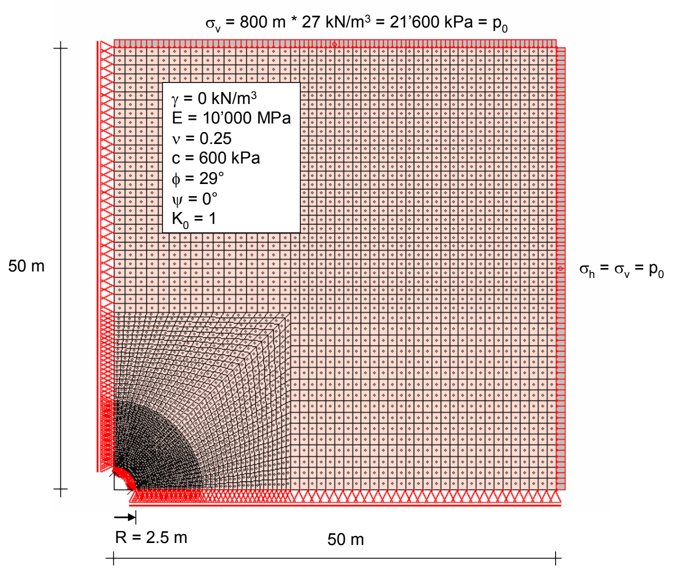

Figure 2. 2D plane strain mesh.

2.2 *Plastic radius and stresses*

The plastic radius R' can be expressed in function of the internal pressure p with the following equation:

$$R' = R\left[\frac{2}{\lambda_p + 1}\,\frac{\sigma_c + p_0(\lambda_p - 1)}{\sigma_c + p(\lambda_p - 1)}\right]^{\frac{1}{\lambda_p - 1}} \qquad (1)$$

with, for the Mohr-Coulomb criterion:

$$\sigma_c = \frac{2c\cdot\cos\phi}{1-\sin\phi} \text{ and } \lambda_p = \frac{1+\sin\phi}{1-\sin\phi}. \qquad (2)$$

Initiation of plastification occurs when R' = R. It occurs theoretically for p = 0.49 p_0, i.e. 51% of unloading. The FE solution identifies the first yielding between 50% and 55% of unloading.

For p = 0, R' = 8.85 m according to (1). Figure 3 illustrates the calculated plastic zones, with R' = 8.875 m.

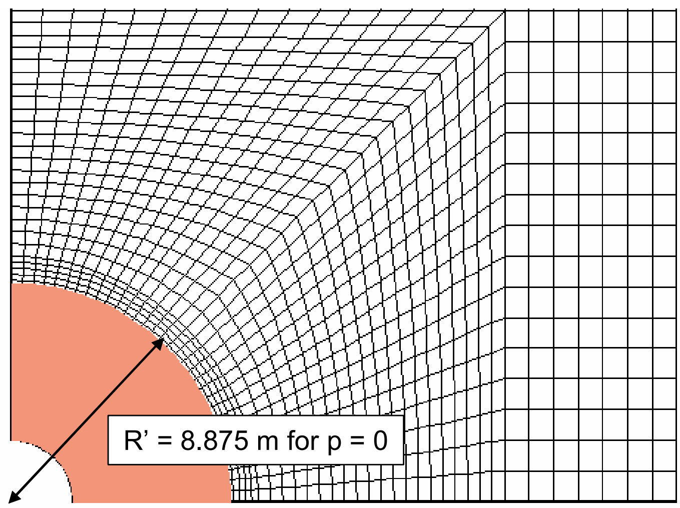

Figure 3. Plastic zones for p = 0.

In the plastic domain, i.e. for R < r < R', the radial and tangential stresses can be expressed as:

$$\sigma_r = \left(\frac{R}{r}\right)^{1-\lambda_p}\left(p + \frac{\sigma_c}{\lambda_p - 1}\right) - \frac{\sigma_c}{\lambda_p - 1} \qquad (3)$$

$$\sigma_\theta = \left(\frac{R}{r}\right)^{1-\lambda_p}\left(p + \frac{\sigma_c}{\lambda_p - 1}\right)\lambda_p - \frac{\sigma_c}{\lambda_p - 1} \qquad (4)$$

In the elastic domain, for r > R':

$$\sigma_r = p_0\left(1 - \frac{R'^2}{r^2}\right) + \sigma_{r=R'}\frac{R'^2}{r^2} \qquad (5)$$

$$\sigma_\theta = p_0\left(1+\frac{R'^2}{r^2}\right)-\sigma_{r=R'}\frac{R'^2}{r^2} \qquad (6)$$

Figure 4 illustrates the good agreement between analytical and numerical distributions of the radial and tangential stresses for p = 0.

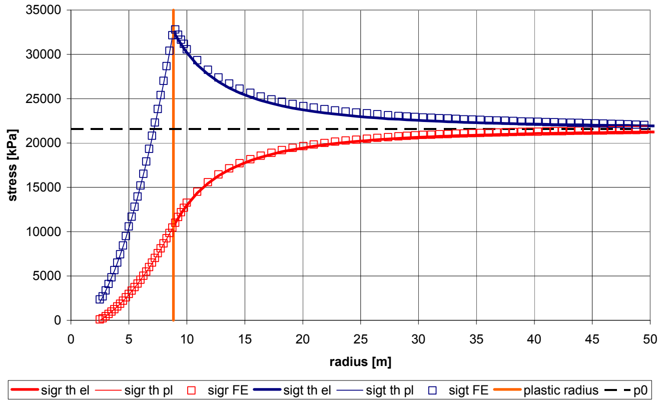

Figure 4. Stresses for p = 0.

2.3 *Displacements*

The convergence curve illustrated in Figure 5 represents the evolution of the radial displacement with respect to the internal pressure.

The maximal difference between the analytical and the numerical solution appears for p = 0, where the theoretical displacement is 61.2 mm and the computed displacement is 63.9 mm.

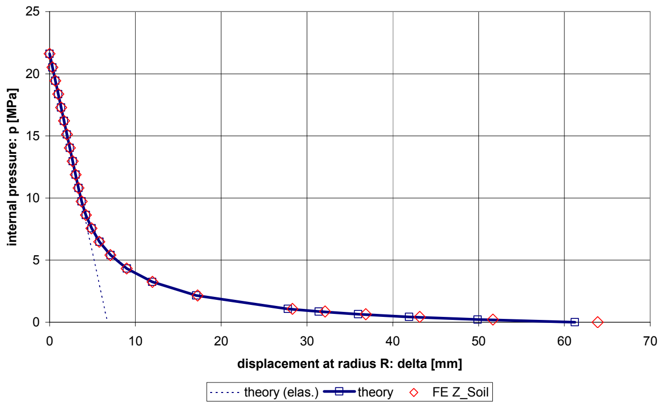

Figure 5. Convergence curve.

2.4 *FE Model with excavation function*

In the example treated above, the internal pressure is explicitly applied on the tunnel's surface as a distributed load, and is driven from p_0 to zero. This can be done only in the hydrostatic case ($K_0 = 1$). In the general case, excavation and unloading can be automatically handled in Z_Soil with the help of an existence and an unloading function, associated with elements which are situated in the tunnel's excavated area. A new computation has been performed, introducing an existence and an unloading function, setting $\gamma = 27$ kN/m^3 in the domain instead of $\gamma = 0$, and $\sigma_v = p_0 - 50$ m * 27 kN/m^3 = 20'250 kPa at the top of the mesh. σ_h varries linearly between 20'250 kPa and 21'600 kPa. Table 1 compares the radial displacements with the analytical solution for both examples. The agreement is very good, as it may be seen.

	Radial displacement [mm]		
p [MPa]	Theory	FE with driven p	FE with exist. func.
21.600	0.0	0.0	0.0
20.520	0.3	0.3	0.3
19.440	0.7	0.7	0.7
18.360	1.0	1.0	1.0
17.280	1.4	1.4	1.3
16.200	1.7	1.7	1.7
15.120	2.0	2.0	2.0
14.040	2.4	2.4	2.4
12.960	2.7	2.7	2.7
11.880	3.0	3.0	3.0
10.800	3.4	3.4	3.4
9.720	3.7	3.7	3.7
8.640	4.2	4.2	4.2
7.560	4.9	4.9	4.9
6.480	5.8	5.8	5.7
5.400	7.1	7.1	7.0
4.320	9.0	9.0	8.9
3.240	12.0	12.0	11.9
2.160	17.1	17.2	17.2
1.080	27.8	28.3	28.3
0.864	31.4	32.1	32.1
0.648	36.0	36.9	36.9
0.432	41.9	43.1	43.0
0.216	49.9	51.7	51.4
0.000	61.2	63.9	63.6

Table 1. Radial displacements.

REFERENCES

Hoek E., Brown T. 1980. Underground excavations in rock, Institution of mining and metallurgy, London

Panet M. 1995. Le calcul des tunnels par la méthode convergence confinement, Presses de l'ENPC, Paris

Z_Soil v7 2007. User manual, www.zace.com, Elmepress

AUTHOR

S. Commend
GeoMod Consulting Engineers

GEOMOD www.geomod.ch   www.zace.com