Aspects of Nonlinear Numerical Simulations for Dam Constructions

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1 Introduction

Numerical analyses of dams may present hidden difficulties in connexion with nonlinear behavior. They require special attention and must be treated carefully. In this paper we show how to handle volumetric locking, a typical problem arising in the simulation of incompressible or dilatant soils and rocks, with the help of a novel approach: stabilized finite element formulations. This approach, already widely used in computational fluid dynamics, has received little attention until now in the field of solid mechanics. It stabilizes the finite element scheme by adding least-square type terms to a mixed displacement-pressure formulation. These terms enhance the stability of the method without compromising consistency, eliminating spurious oscillations in the pressure field in single- as well as two-phase media. In the first part of the paper, issues related to volumetric locking are presented, as well as popular techniques used to handle this problem. The shortcomings of the standard displacement finite element formulation are illustrated with a superficial footing bearing capacity problem. We also show how the stabilized approach can avoid volumetric locking in this case. In the second part, we show how to apply the stabilized approach to an elastoplastic continuum. We start with the governing equations of the mixed displacement-pressure formulation, and we define the stabilizing terms which will be added to the matricial form of the problem. The genericity and the robustness of the method is also addressed. Three real-world applications are then presented in order to validate the approach: a two-phase consolidation analysis of an earth-dam where spurious oscillations in the pore pressure field are shown to disappear in the stabilized case, and two numerical analyses of concrete dams, including an integrated approach of a three-dimensional case. Finally, some conclusions are drawn.

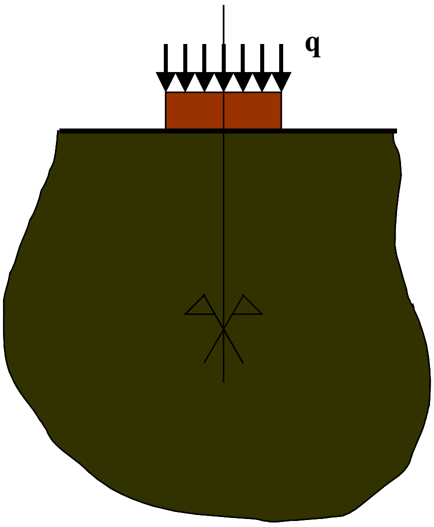
2 Issues related to volumetric locking

2.1 Element technology

As standard finite elements show a tendency to lock in incompressible or dilatant elastoplastic cases, investigations have been made in order to find low-order elements which do not lock. Hughes [1] introduced the **B-bar** approach in the early eighties, modifying the volumetric contribution of numerically computed strains. Averaging or underintegration techniques are shown also to handle the incompressible case correctly. However, the **B-bar** formulation does not exist for linear 3-node triangles (T3) and the performance of this technique in dilatant plastic flows has been criticized [2]. In the same paper, another method is analyzed: Enhanced Assumed Strains (EAS) elements, originally proposed by Simo and Rifai [3]. EAS elements are shown to yield satisfactory results in both the incompressible and the dilatant case. But again the lack of such a formulation for a simple element like the linear triangle (or tetrahedron in 3D) can represent a shortcoming in practical modelling cases. We will show in the next section how we build a generic stabilized mixed formulation which allows the mixture of different element types in the same mesh and is valid for any plastic flow.

2.2 Illustration of volumetric locking

But first let us illustrate with a practical case what is meant by volumetric locking. Consider the bearing capacity of the superfical footing example illustrated in Figure 1. We wish to find the ultimate load that this footing can carry. Following Terzaghi [4] or Matar & Salençon [5], the theoretical failure should occur for $q = 16 \rightarrow 18$ kN/m. If we plot the solution of the standard finite element displacement formulation (Fig. 2) considering a dilatant soil material, we notice a strong overshoot of the limit load: this formulation cannot correctly predict this limit. This phenomenon is typical of volumetric locking. In the same graph, we see that both the EAS formulation and the stabilized approach give a more reasonable approximation of the failure load. We will describe next how to construct such a stabilized formulation.



$E = 3000$ kN/m^2 ; $\nu = 0.38$; $\gamma = 0$

$c = 1$ kN/m^2 ; $\phi = 20°$; $\psi = 20°$
Drucker-Prager material, with a plane-strain adj. on Mohr-Coulomb

Surface load $\mathbf{q} = 0 \rightarrow 20$ kN/m

$\mathbf{q} = 18$ kN/m^2 corresponds to failure [4]

Figure 1 – Superficial footing characteristics

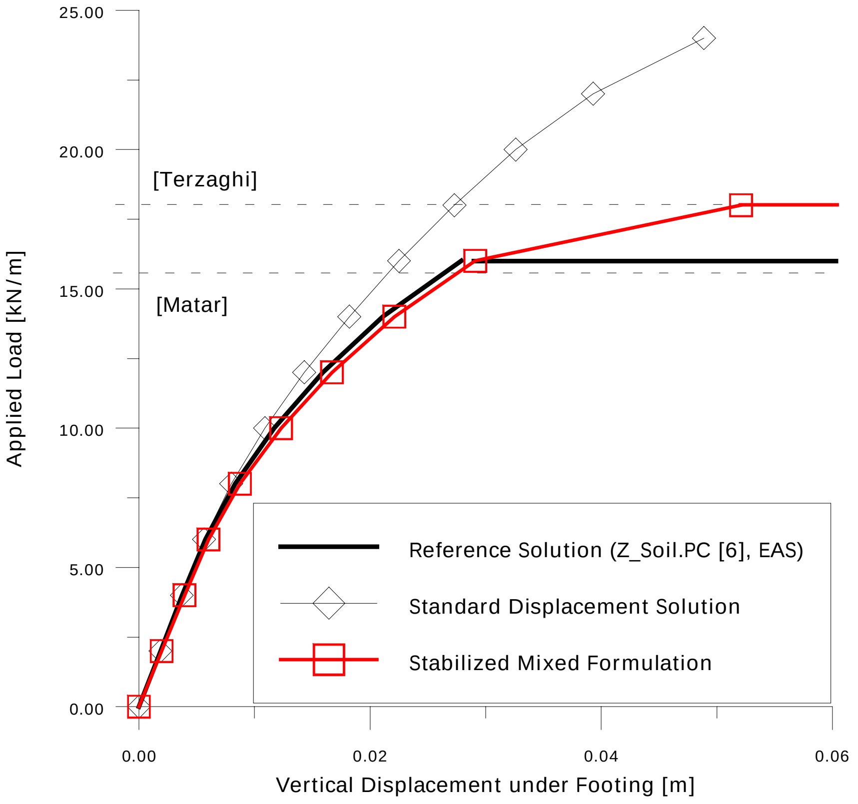


Figure 2 – Vertical displacement under the footing

3 The stabilized approach

3.1 Governing equations of the single phase problem

We start with the classical static equilibrium equation:

$$div(\boldsymbol{\sigma}) + \mathbf{f} = \mathbf{0}$$
(1)

where is the stress tensor and **f** the external load vector. In standard elastoplastic finite element formulations, the stress is computed from stress increments . The incremental constitutive relation then reads:

$$\boldsymbol{\Delta\sigma}(\boldsymbol{\Delta u}) = \mathbf{D}(\boldsymbol{\Delta\varepsilon}(\boldsymbol{\Delta u}) - \boldsymbol{\Delta\varepsilon}^p(\boldsymbol{\Delta u}))$$
(2)

with **D** the elastic modulus matrix, $\boldsymbol{\Delta\varepsilon}$ the total strain increment and $\boldsymbol{\Delta\varepsilon}^p$ the plastic strain increment. The resolution procedure is based on a discretized weak form in terms of displacement incrementts **u**. The problem has to be solved iteratively with the help of a Newton-Raphson like scheme as we deal with a nonlinear problem.

A mixed displacement-pressure form is obtained by introducing the following volumetric-deviatoric split into the constitutive equation for the stress increment (details of the matrix form are given in appendix A):

$$\boldsymbol{\Delta\sigma}(\boldsymbol{\Delta u}, \Delta p) = \overline{\mathbf{D}}(\boldsymbol{\Delta\varepsilon}(\boldsymbol{\Delta u}) - \boldsymbol{\Delta\varepsilon}^p(\boldsymbol{\Delta u}, \Delta p)) + \mathbf{1}\Delta p$$
(3)

3.2 *Towards a mixed matricial formulation*

Consider the reference problem illustrated in Figure 3. The strong form of the corresponding mixed elastoplastic boundary value problem can be stated as: find the displacement **u** and the pressure p such that:

$div(\boldsymbol{\sigma}) + \mathbf{f} = \mathbf{0}$ Equilibrium equation over the domain

(4)

$div(\mathbf{u}^e) - \frac{p}{K} = 0$ Pressure constitutive equation over the domain

(5)

$\mathbf{u} = \mathbf{g}$ Displacement boundary condition on $_g$

(6)

$\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{h}$ Stress boundary condition on $_h$

(7)

where the evolution of the stress is governed by the incremental constitutive equation (3).

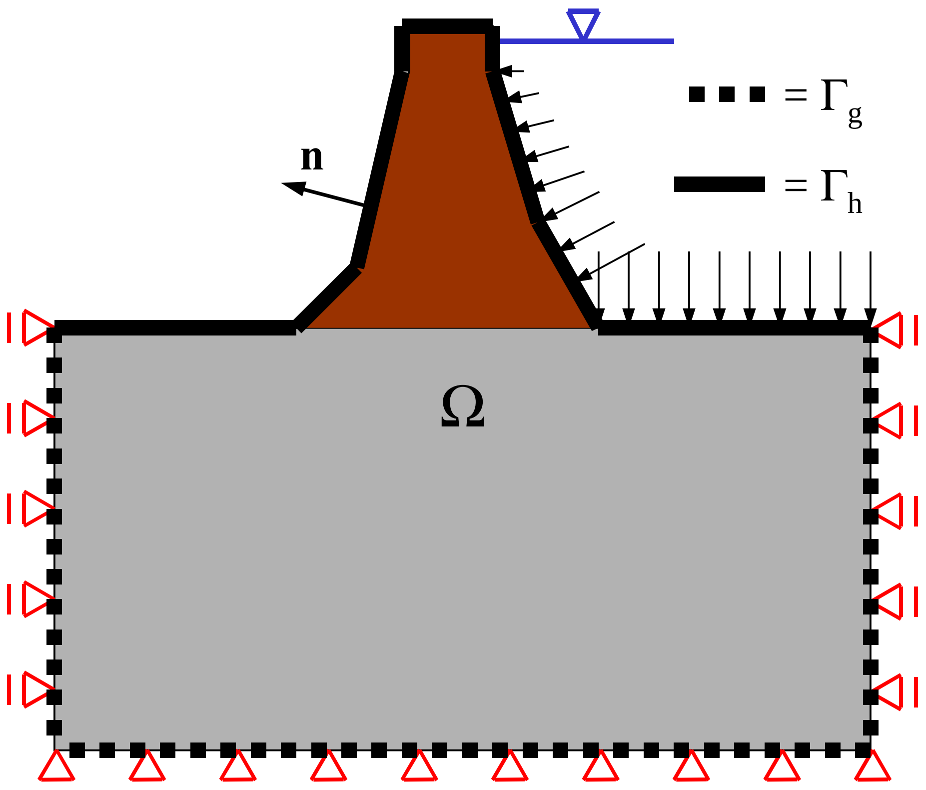


Figure 3 – Reference boundary value problem

A corresponding weak form is constructed by multiplying Equations (4) and (5) by appropriate weighting functions. Integrating by parts and making use of the natural boundary condition leads to a variational formulation equivalent to Equations (4)-(7). Introducing shape functions, we recover a discretized (or matricial) form of the boundary value problem that we can solve for **u** and **p**:

$$\begin{bmatrix} \mathbf{K_{uu}} & \mathbf{K_{up}} \\ \mathbf{K_{pu}} & \mathbf{K_{pp}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{u} \\ \Delta\mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{F_u} \\ \mathbf{F_p} \end{bmatrix}$$

(8)

Unfortunately, the convergence of the described mixed approach is known to be dependent on the interpolation adopted for displacements and pressures and a stabilization technique is necessary to avoid spurious oscillations in the pressure field.

3.3 Stabilization technique

Following the approach applied by Hughes et al. to computational fluid dynamics [7-8], we consider adding to the system of equations (8) stabilizing terms of the form:

$$\sum_{e=1}^{n_{el}} \int_{\Omega^e} \underline{\left[\mathbf{L}^T \boldsymbol{\sigma}(\mathbf{w}^h, q^h)\right]}^T \boldsymbol{\tau} \left[\mathbf{L}^T \boldsymbol{\sigma}(\mathbf{u}^h, p^h) + \mathbf{f}\right] d\Omega$$

(9)

which is briefly discussed in appendix B and in details in [9]. In this formulation, the notion of stabilization factor has been introduced. Following numerous benchmarks, it has been found that a factor of the type:

$$\boldsymbol{\tau} = \tau \cdot \mathbf{I} = \frac{\alpha^e \left(h^e\right)^2}{2\mu} \cdot \mathbf{I}$$

(10)

comparable to what Hughes proposed for incompressible Stokes flow [7], gives satisfactory results also in the elastoplastic case. h^e is a characteristic length of the considered element (for instance the square root of its volume), is the material's shear modulus and ^{e} is a scalar parameter which should be chosen in the lower range of the interval [10^{-1}; 1].

3.4 Genericity and robustness of the stabilized approach

We have seen in section 2 classical techniques that are known to cure the shortcomings of the standard finite element displacement formulation. The stabilized approach introduces two important advantages when compared to these formulations:

- it is generic, i.e. it applies to any type of element (in two or three dimensions, regardless of the number of nodes or the respective order of interpolation of both fields)

- it is robust, i.e. it becomes possible to mix different kinds of elements in the same mesh. Consider for instance Figure 4 where we have represented the solution of a driven cavity flow test reproduced for an incompressible elastic material. Comparing the results obtained with a mixture of **B-bar** Q4 and standard T3 with the ones given by the stabilized approach allows us to conclude that the stabilized scheme is more robust.

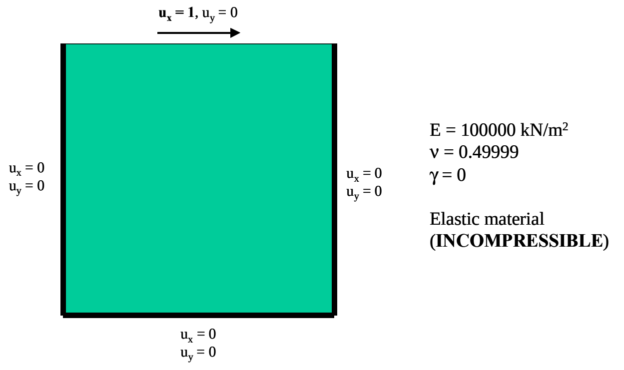


Figure 4a – The problem

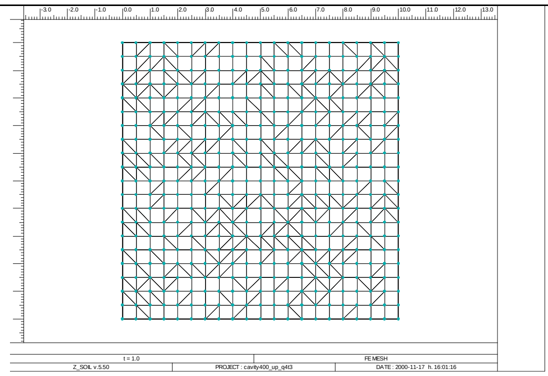


Figure 4b – The finite element mesh

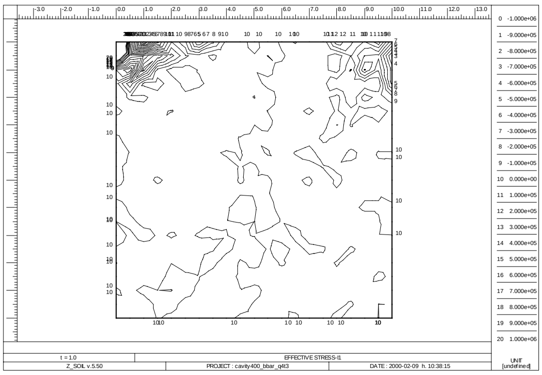


Figure 4c – Pressure field isolines
(Q4 **B-bar** + standard T3)

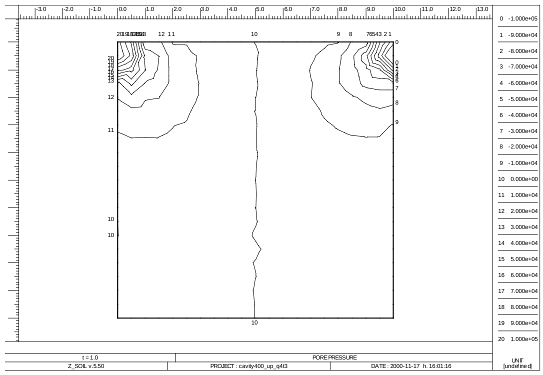


Figure 4d – Pressure field isolines
(Stabilized Q4 and T3)

Figure 4 – The driven cavity flow example

3.5 Extension to two-phase problems

Spurious oscillations in the pore pressure field can also be eliminated by means of the same kind of stabilization technique, but this time stabilizing terms are derived from the residuum of the fluid flow continuity equation [10]. Among other benefits, this formulation circumvents the lower limit time step condition that must be fulfilled to get non-oscillatory solutions when the standard formulation is used [11]. This is illustrated next.

4 Applications

4.1 Consolidation analysis under an earth-dam

The stabilization technique described in [10] is used here to compute the pore pressure redistribution after construction of an earth-dam. Color maps of the pore overpressure are represented for two analyses: the first one using a standard formulation and the second a stabilized formulation. The calculation is performed as follows: first, an undeformed initial state is computed without the dam, then construction stages are taken into account and the pore overpressure develops under the dam. In the early stages of the analyses, the time step must be chosen to be small and violates the time step condition; therefore the pore pressure solution obtained with the standard analysis shows useless oscillatory pressure patterns, while the stabilized pressure field does not suffer from such a drawback for the same time stepping (see Figure 5).

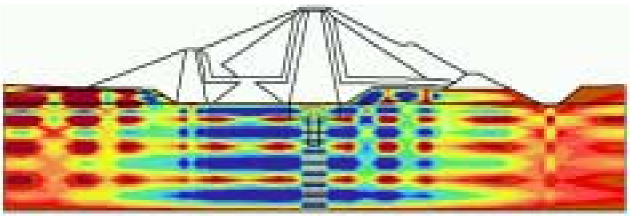


Figure 5a – Standard formulation solution

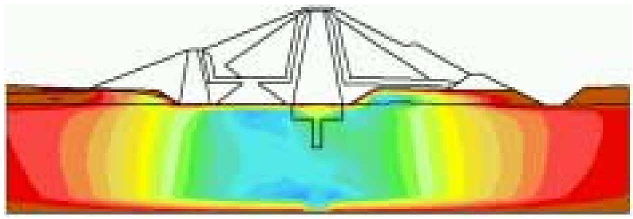


Figure 5b – Stabilized approach solution

Figure 5 – Pore pressure color maps

4.2 A concrete dam analysis

A safety analysis of a concrete dam has been conducted [12]. The geometry of the dam is illustrated in Figure 6a. The presence of different materials which may exhibit incompressible or dilatant behavior (and even softening in the concrete) does not allow us to use standard finite elements: although volumetric locking is more difficult to notice here than in the superficial footing case, the possibility of such a phenomenon cannot be ruled out and a proper finite element formulation must be used. That is why we handle this computation with the help of stabilized elements. As an illustration of the results, the development of plastic (in red) and unloaded (in green) zones in the dam when the water level has reached its crown is given in Figure 6b.

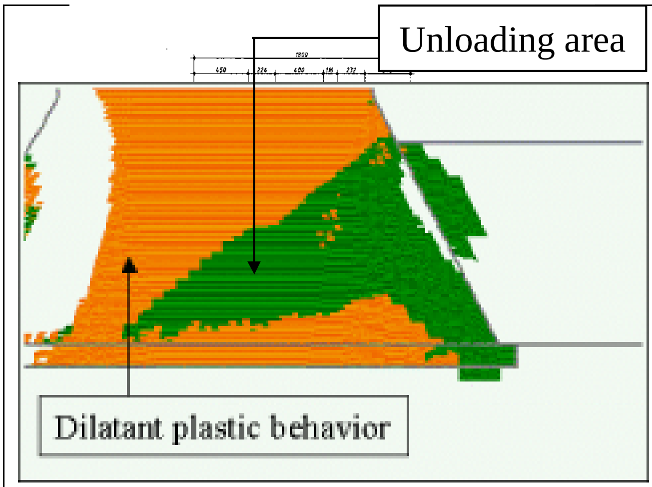


Figure 6a – Dam geometry and properties

Figure 6b – Detail A: plastic zones

Figure 6 – A concrete dam study

4.3 The Shahid Rajaee dam case study (by Stucky Consulting Engineers Ltd)

The Shahid Rajaee dam is a concrete arch structure impounding a reservoir with a capacity of 190 million m^3. The dam foundation is heterogeneous. Whereas the dam toe is located on marly sandstone with low compressive strength ($_c$ ~12.5 MPa), the flanks are founded on sound sandstone ($_c$ ~7.0 MPa) and the upper arches are abuted against limestone ($_c$ ~85 MPa). The pronounced heterogeneity of the foundation strength and also its deformability necessitated the simulation of the induced displacements in order to compare them with in situ measures. Figure 7 shows a general view of the structure and the finite element mesh used for numerical simulation, while Figure 8 shows the comparison of in situ measured displacements with calculated values for the numerical model. There again a locking-free formulation is essential to an accurate prediction of the displacements.

Figure 7a – Construction of the dam

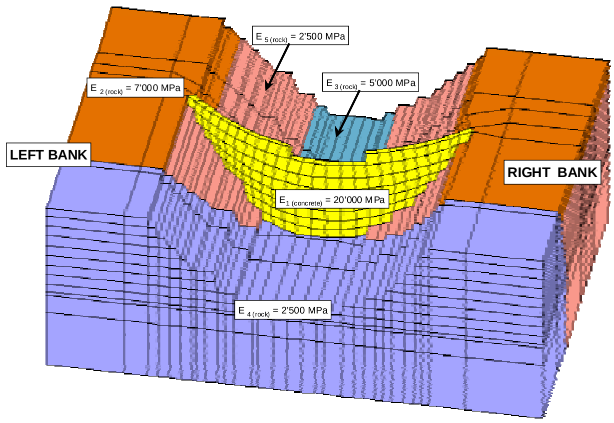


Figure 7b – 3D finite element mesh

Figure 7 – The Shahid Rajaee dam

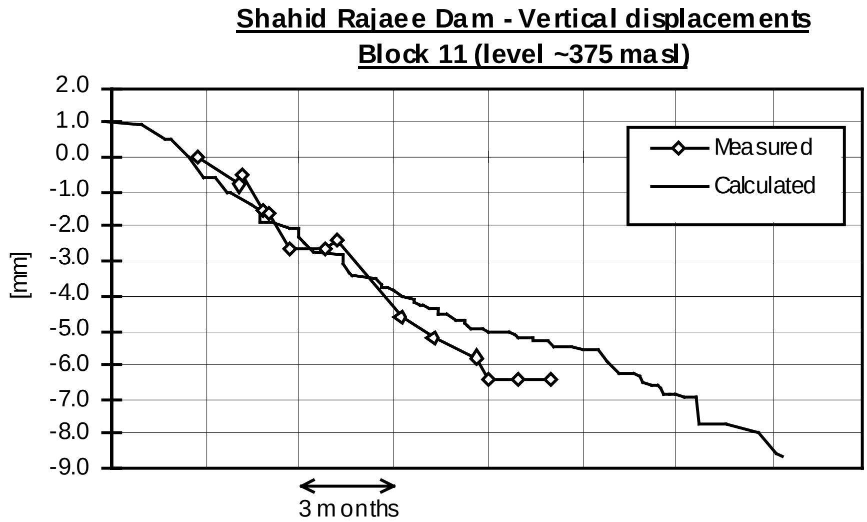


Figure 8 – Displacements comparison

Conclusions

Numerical simulations of the behavior of hydraulic structures often involve incompressible or dilatant materials in single- or two-phase media. These materials require special finite element formulations in order to avoid spurious results. We propose in this paper a novel approach to overcoming these problems which is simple to implement, generic and robust. Several applications to full size hydraulic structures illustrate the efficiency of the novel approach.

Appendix A

Consider the following incremental constitutive equation (corresponding to Equation (3)):

$$\boldsymbol{\Delta\sigma}(\boldsymbol{\Delta u}, \Delta p) = \overline{\mathbf{D}}(\boldsymbol{\Delta\varepsilon}(\boldsymbol{\Delta u}) - \boldsymbol{\Delta\varepsilon}^{p}(\boldsymbol{\Delta u}, \Delta p)) + \mathbf{1}\Delta p$$
(11)

p, the hydrostatic pressure increment in the solid phase, can be expressed as:

$$\Delta p = K\mathbf{1}^{T}(\boldsymbol{\Delta\varepsilon}(\boldsymbol{\Delta u}) - \boldsymbol{\Delta\varepsilon}^{p}(\boldsymbol{\Delta u}, \Delta p))$$
(12)

In Equation (11), the deviatoric projection of **D** has been introduced:

$$\overline{\mathbf{D}} = \mathbf{D}(\mathbf{I} - \frac{1}{3}\mathbf{1}\mathbf{1}^{T})$$
(13)

where **I** is the identity matrix and **1** the vectorial representation of Kronecker's delta $_{ij}$. Finally, K is the material's elastic bulk modulus:

$$K = \frac{E}{3(1-2\nu)}$$
(14)

with E the material's Young modulus and its Poisson ratio.

Appendix B

Stabilizing terms take the general form:

$$\sum_{e=1}^{n_{el}} \int_{\Omega^{e}} \underline{\left[\mathbf{L}^{T}\boldsymbol{\sigma}(\mathbf{w}^{h}, q^{h})\right]}^{T} \boldsymbol{\tau} \left[\mathbf{L}^{T}\boldsymbol{\sigma}(\mathbf{u}^{h}, p^{h}) + \mathbf{f}\right] d\Omega$$
(15)

where **L** is a differential operator which has the effect of taking the divergence of . The $(.)^{h}$ superscript indicates discretized values, while **w** and q are weighting functions corresponding respectively to **u** and p. In the range of studied cases, experience has shown that neglecting the contribution of $\mathbf{w}^{h}$ in the underlined weighting part of Equation (15) leads to results which are, on the one hand, at least as accurate as the ones obtained with the full Galerkin Least-Squares scheme and, on the other hand, easier to produce. In this case the stabilizing terms reduce to:

$$\sum_{e=1}^{n_{el}} \int_{\Omega^e} \underline{\nabla N^{p}}^{T} \, \boldsymbol{\tau} \left[\mathbf{L}^T \boldsymbol{\sigma}(\mathbf{u}^h, p^h) + \mathbf{f} \right] d\Omega$$
(16)

where N^p are pressure shape functions. This type of stabilization that we have called “modified-GLS” is the one that we have used in all the subsequent analyses. Another possibility, first studied in the field of computational soil mechanics by Pastor et al. [13], is to consider adding terms of the form:

$$\sum_{e=1}^{n_{el}} \int_{\Omega^e} \underline{\nabla N^{p}}^{T} \, \boldsymbol{\tau} \nabla N^{p} \, d\Omega$$
(17)

This latter technique has shown very similar results when compared to the “modified-GLS” scheme in the benchmarks that we have produced. Finally, a directional feature can be introduced in the stabilization terms, following the finite increment calculus formulation described by Oñate [14]. However, the advantages of this approach have yet to be discovered. Details on how this directional formulation provides a tentative physical justification to the “modified-GLS” scheme can be found in [9].

Acknowledgements

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