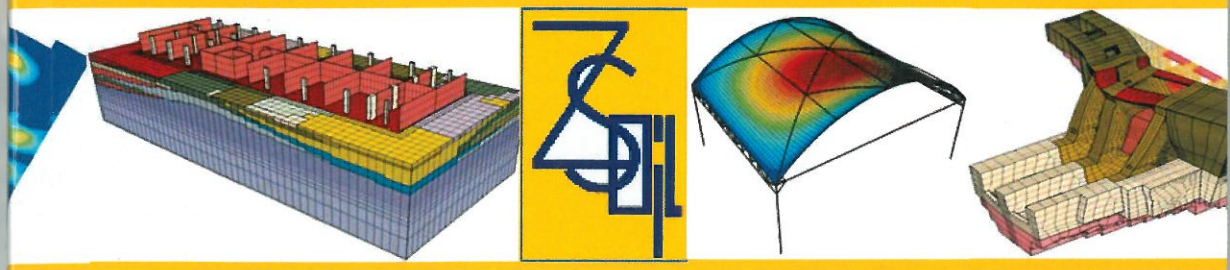

NUMERICS IN GEOTECHNICS AND STRUCTURES 2010

Edited by
Th. Zimmermann
A. Truty
K. Podleś

Elmepress International

On the use of the Hardening Soil Small Strain model in geotechnical practice

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Keywords: Deep excavation, retaining wall, tunnel excavation, advanced soil model

Abstract

This article highlights the importance of using advanced constitutive soil models in numerical modelling as daily geotechnical practice. In this context, the Hardening Soil (HS) model is presented as the most advanced soil model implemented in the ZSoil finite element code. The article recalls the basic features of the HS model and it explains their role in numerical simulations. The role of pre-failure non-linear behaviour of soils is also discussed. An application of the HS model is illustrated on typical geotechnical problems such as retaining wall excavation, tunnel excavation and undrained loading followed by consolidation.

1. Introduction

The use of the finite element (FE) analysis has become widespread and popular in geotechnical practice as means of controlling and optimizing engineering tasks. However, the quality of any stress-strain prediction depends on the adequate model being adopted in the study. In general, a more realistic prediction of ground movements requires using the models which account for pre-failure behaviour of soil, i.e. a non-linear stress-strain relationship before reaching the ultimate state (cf. [1]). Such behaviour, mathematically modelled with non-linear elasticity, is characterized by a strong variation of soil stiffness which depends on the magnitude of strain levels occurring during construction stages. Pre-failure stiffness plays a crucial role in modelling typical geotechnical problems such as deep excavations supported by retaining walls or tunnel excavations in densely built-up urban areas.

It is commonly known that soil behaviour is not as simple as its prediction with simply-formulated linear constitutive models, which are commonly and carelessly used in numerical analyses. Complex soil behaviour which stems from the nature of the multi-phase material exhibits both elastic and plastic non-linearities. Deformations include irreversible plastic strains. Depending on the history of loading, soil may compact or dilate, its stiffness may depend on the magnitude of stress levels, soil deformations are time-dependent, etc. In fact, soil behaviour is considered to be truly elastic in the range of small strains as schematically presented in Figure 1. In this range of strain, soil may exhibit a nonlinear stress-strain relationship. However, its stiffness is almost fully recoverable in unloading conditions. Following of pre-failure non-linearities of soil behaviour, one may observe a strong variation of stiffness starting from very small shear strains, which cannot be reproduced by models such as the linear-elastic Mohr-Coulomb model.

Engineers who are looking for **reliable and realistic predictions** of the engineering system response should be aware that by applying linear-elastic, perfectly plastic models in

the FE analysis, soil ground movements may be underestimated, which may influence the magnitude of forces which are computed for supporting structural elements. The models which account for high stiffness at very small strains concentrate the development of high amplitudes of strain around the close neighbourhood of the source of deformations, similarly to what is observed in reality. This can be the case of **retaining walls** or **tunnel excavations** where soil stiffness degrades increasing soil deformations in the close vicinity of unloaded boundaries, and appropriately reducing them away from the unloaded zone (cf. [2]).

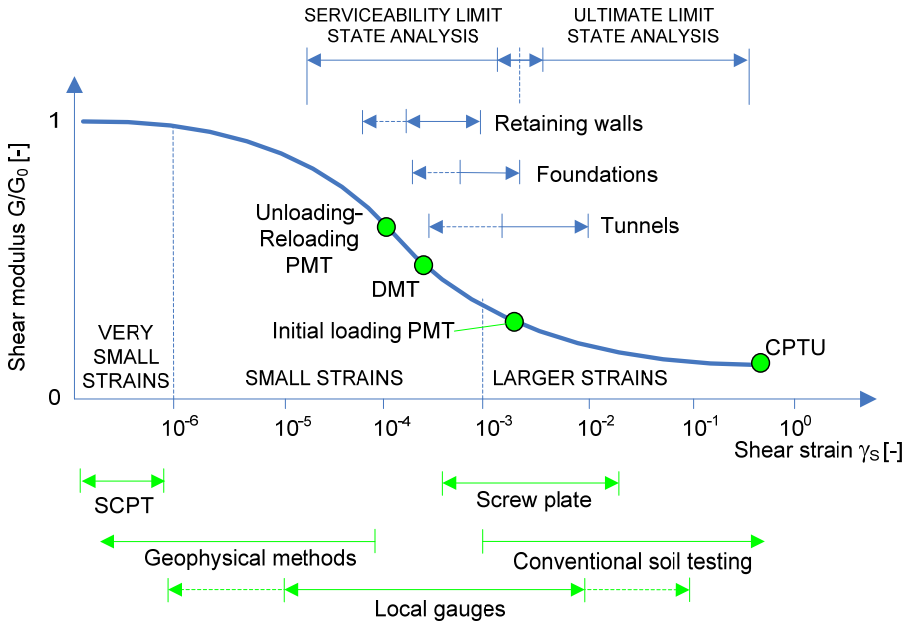


Figure 1 Typical representation of stiffness variation in as a function of the shear strain amplitudes; comparison with the ranges for typical geotechnical problems and different tests (based on [1] and updated by the author); SCPT - seismic cone penetration test; CPTU - piezocone penetration test; DMT - Marchetti's dilatometer test; PMT - Pressuremeter test.

The **Hardening Soil** (HS) model in its two variants HS-Standard and HS-SmallStrain can be a solution for modelling of the problems which have been listed above, as they account for most of soil behaviour features (see Section 3). Despite the mathematical complexity of the HS model, its **parameters have explicit physical meaning and can be determined with conventional soil tests** [3]. The present article presents a few examples of the application of this constitutive model for typical geotechnical problems.

2. Choice of the constitutive model

The finite element code ZSoil® includes a variety of soil models from simple linear elastic, perfectly plastic (e.g. Mohr Coulomb), elasto-plastic cap models (e.g. Cap, Modified Cam Clay) to advanced nonlinear-elasto-plastic cap model HS-SmallStrain. The choice of a constitutive model depends on many factors but, in general, it is related to the type of analysis that the user intends to perform, expected precision of predictions and available knowledge of soil.

As regards the type of analysis, geoengineering computings can be divided into two groups (see Figure 2): (a) those whose goal is to assess bearing capacity and slope or wall stability which are related to the ultimate limit state analysis (ULS), and (b) those which are related to the limit state analysis (SLS), such as deep excavations or tunnel excavations in urban areas.

In general, as long as assessment of ULS for bearing capacity or slope stability is foreseen, the analysis may be limited to basic linear models such as the Mohr-Coulomb model (but this is not a rule). On the other hand, a precise deformation analysis requires the application of advanced constitutive models which approximate the stress-strain relation more accurately than simple linear-elastic, perfectly plastic model, and in effect, the form of displacement fields can be modelled more realistically.

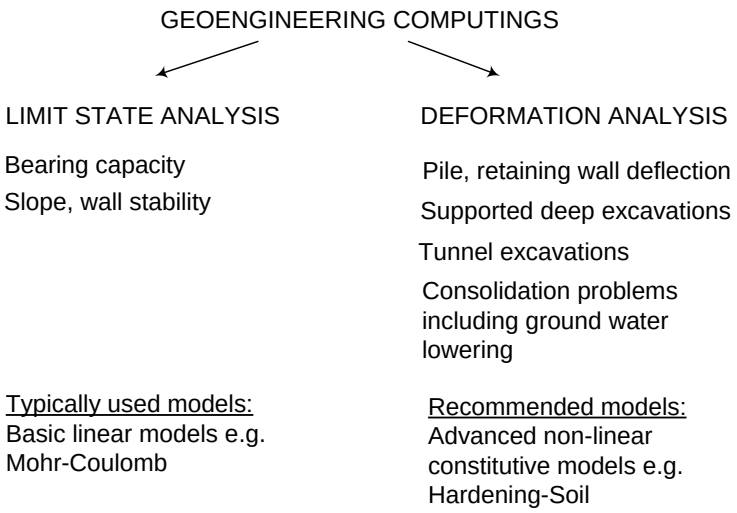


Figure 2 General types of geoengineering computings.

The Hardening-Soil model realistically reproduces soil deformations, as the σ-ε relation is approximated with a non-linear curve (the hyperbolic function by Duncan-Chang, for details see [7],[3]). Moreover, as the formulation of the HS model incorporates two hardening mechanisms, it is suitable for modelling both domination of shear plastic strains which can be observed in granular soils and in overconsolidated cohesive soils, as well as domination of compressive plastic strains which is typical for soft soils, see Figure 3.

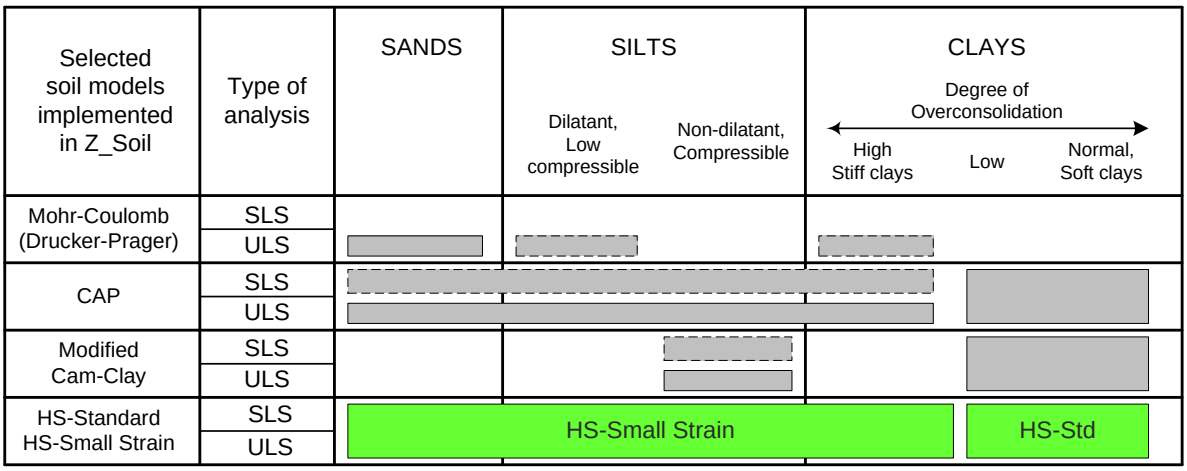


Figure 3 Recommendations for the model choice for soil type and types of analysis. Dashed line: may be used but not recommended in terms of quality of results; Solid line: can be applied; Green fill: recommended.

3. A short overview of the Hardening Soil model

3.1 Features of the model

The Hardening Soil model (HS-Standard) was designed by [2], [5] in order to reproduce basic macroscopic phenomena exhibited by soils such as:

- **densification**, i.e. a decrease of voids volume in soil due to plastic deformations,
- **stress dependent stiffness**, i.e. commonly observed phenomena of increasing stiffness modules with increasing confining stress (also related to increasing depth);
- **soil stress history**, i.e. accounting for preconsolidation effects;
- **plastic yielding**, i.e. development of irreversible strains with reaching a yield criterion;
- **dilatancy**, i.e. an occurrence of negative volumetric strains during shearing.

Contrary to other models such us the Cap model or the Modified Cam Clay (let alone the Mohr-Coulmb model), the magnitude of soil deformations can be modelled more accurately by incorporating three different input stiffness parameters which correspond to the triaxial loading stiffness (E_{50}), the triaxial unloading-reloading stiffness (E_{ur}), and the oedometer loading modulus (E_{oed}).

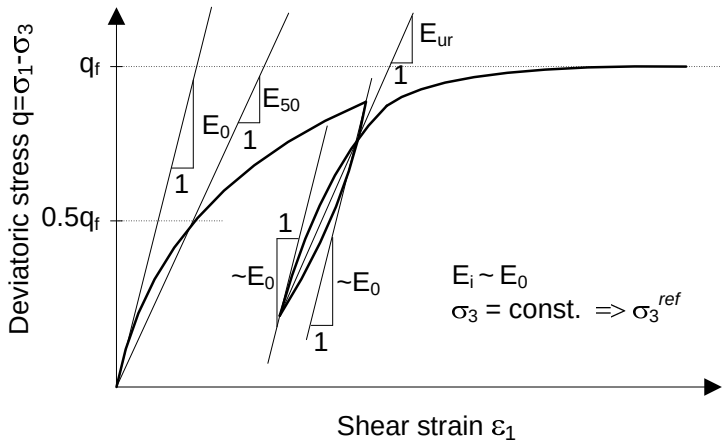


Figure 4 Common definitions of different moduli on a typical strain-stress curve for soil.

An enhanced version of the HS-Standard, the Hardening Soil Small model (HSSmallStrain) was formulated by Benz [6] in order to handle the commonly observed phenomena of:

- **strong stiffness variation** with increasing shear strain amplitudes in the domain of small strains (S-shape curve presented in Figure 1);
- **hysteretic, nonlinear elastic stress-strain relationship** which is applicable in the range of small strains.

These features mean that the HS-SmallStrain is able to produce more accurate and reliable approximation of displacements which can be useful for dynamic applications or in modelling unloading-conditioned problems, e.g. **excavations with retaining walls** or **tunnel excavations**.

A detailed description of the Hardening Soil model can be found in ZSoil reports [3][7].

3.2 Hardening mechanisms and their role in FE simulations

In the Hardening Soil model, accounting for the history of stress paths is possible thanks to **two hardening mechanisms**, i.e. *isotropic* and *deviatoric*. The first one, the volumetric plastic mechanism in the form of cap surface is introduced to account for a threshold point (**preconsolidation pressure**) beyond which important plastic straining occurs, characterizing a normally-consolidated state of soil. Since the shear mechanism generates no volumetric plastic strain in the contractant domain, the model without volumetric mechanism could significantly overestimate soil stiffness in virgin compression conditions, particularly for normally- and lightly overconsolidated cohesive soils. Such a problem can be observed when using, for instance, the linear elastic-perfectly plastic *Mohr-Coulomb model*. The isotropic mechanism is thus very important when **modelling consolidation problems related to footing or ground water lowering** (see Figure 5a). In the case of a footing problem, the HS model may generate non-linear relation ε_1-q before reaching the ultimate state even for a purely deviatoric stress path (e.g. loading in undrained conditions). Note that the ultimate state in the HS model is defined by the *Mohr-Coulomb criterion*. In the case of normally- and lightly overconsolidated soils two hardening mechanisms may be activated by footing-related stress paths, resulting in important plastic straining.

Another feature of the ***cap mechanism*** is that it enables a degradation of soil stiffness (i.e. secant modulus E) with the increasing level of strain (see Figure 6b). This feature can be useful for modelling unloading modes of soil, due to excavation for instance. It is often observed in numerical analyses that not differentiating between loading and unloading stiffness modules in the Mohr-Coulomb model may result in an unrealistic lifting of the retaining wall associated with unloading of the bottom of an excavation. A combination of input parameters for the Cap model allows the user to distinguish between *loading* and *unloading-reloading modules* for which a typical ratio is around $E_{ur}/E \approx 3 - 10$ as the ratio for compression indices C_c / C_s is typically measured in oedometric tests between 0.1 and 0.4. In the Cap model, the input modulus E can be considered as the unloading-reloading modulus E_{ur} and the slope of the normal consolidation line λ which controls the magnitude of current soil stiffness for a given vertical stress defines the initial loading modulus E, assuming that a material is in normally-consolidated state (note that for OCR = 1, the initial stress point is located on the normal consolidation line):

$$E = \frac{(1+\upsilon)(1-2\upsilon)}{(1-\upsilon)} E_{oed}$$

where

$$E_{oed} = \frac{2.303(1+e^{ref})}{C_c} \sigma_{oed}^{ref} \approx \frac{(1+e^{ref})}{\lambda} \sigma_{oed}^{ref}$$

with E_{oed} denoting a *tangent oedometric modulus* which corresponds to a given reference oedometric (vertical) stress, and ν is the Poisson's ratio.

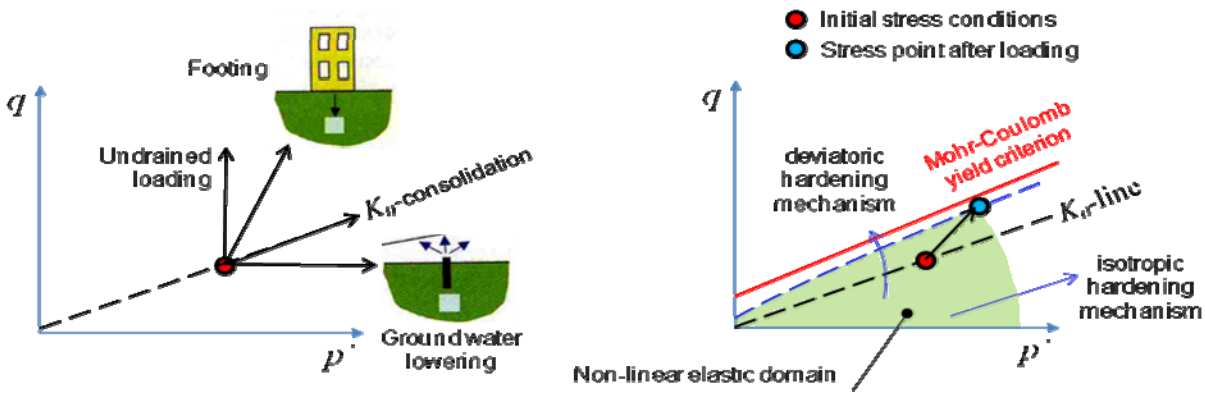


Figure 5 Typical stress paths related to footing and ground water lowering (on the left) and activation of both hardening mechanisms in the Hardening-Soil model for a footing type stress path (on the right).

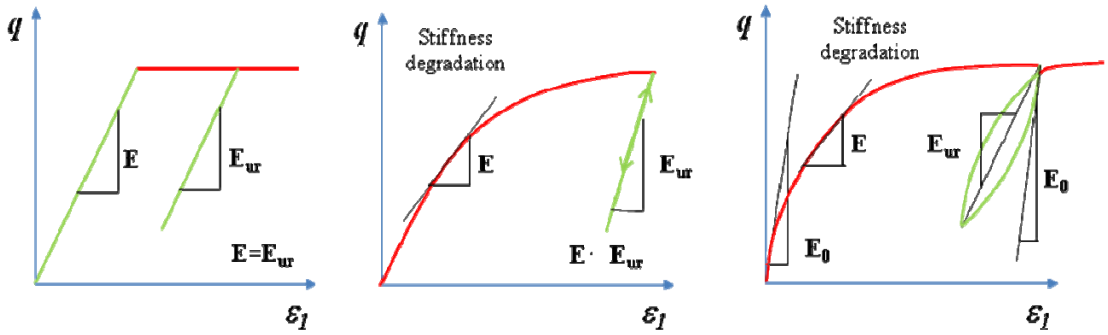


Figure 6 Typical stress-strain curves obtained from triaxial compression test with different constitutive models: (left) Mohr-Coulomb model, (center) Cap model linear elasticity (at the beginning of ε_1-q curve) for lightly-overconsolidated material, (right) Hardening Soil-SmallStrain model with nonlinear elasticity for lightly-overconsolidated material.

With such a simplified approach, the user may reduce the problem of exaggerated swelling of the excavation bottom, but there is still a problem with soil elements which are located behind the retaining wall. Using the Cap model which is characterized by linear elasticity, soil stiffness behind the wall may be overestimated because the stress paths before yielding remain in the linear elastic domain (refer to the direction of the stress paths for a soil element behind the retaining wall in Figure 7a). Non-linear ε–σ relationship before yielding can be modelled using the Hardening Soil-Standard or SmallStrain models thanks to the deviatoric hardening mechanism.

The ***shear (deviatoric) mechanism*** is introduced in order to handle the soil hardening which is induced by the plastic shear strains. The domination of plastic shear strains can be typically observed for granular materials such as sands and heavily consolidated cohesive soils. One may also expect domination of plastic shear strains for soil elements behind a retaining wall (see stress paths in Figure 7a) as it can be concluded from observations of soil settlements which may occur behind a retaining structure. Note that when using constitutive models such as Mohr-Coulomb or the Cap model, soil response behind the retaining wall is linearly elastic during excavation, and thus horizontal displacements and wall deflection may be underestimated (see Figure 8a).

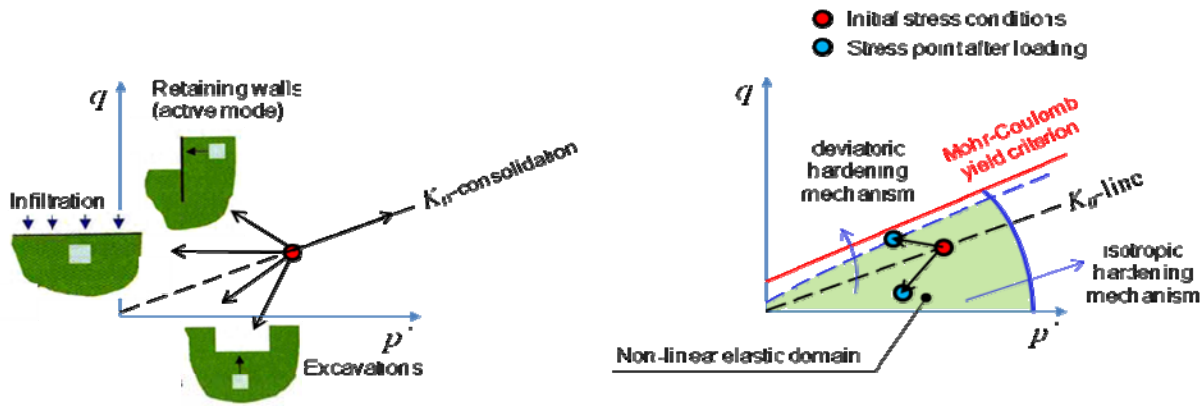


Figure 7 Typical stress paths related to infiltration and excavation problems (on the left) and activation of the deviatoric hardening mechanisms and unloading modes for excavation-related stress paths (on the right).

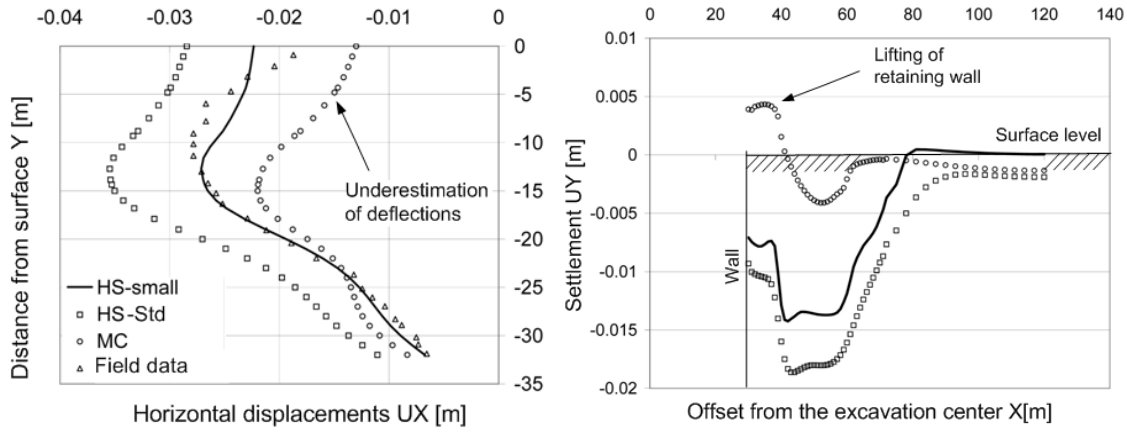


Figure 8 Example of a deep excavation in Berlin Sand (for details see [2]). Comparison of model predictions and *in situ* data: wall deflections (left), surface settlements (right).

3.3 Model limitations

Although the HS model can be considered an advanced soil model which is able to faithfully approximate complex soil behaviour, it includes some limitations related to specific behaviour observed for certain soils. The models are not able to reproduce softening effects associated with soil dilatancy and soil destructuration (debonding of cemented particles) which can be observed, for instance, in sensitive soils.

As opposed to the HS-SmallStrain model, the HS-Standard does not account for large amplitudes of soil stiffness related to transition from very small strain to engineering strain levels ($\varepsilon \approx 10^{-3}$–$10^{-2}$). Therefore, the user should adapt the stiffness characteristics to the strain levels which are expected to take place in conditions of the analyzed problem. Moreover, the HS-Standard model is not capable to reproduce hysteretic soil behaviour observed during cycling loading.

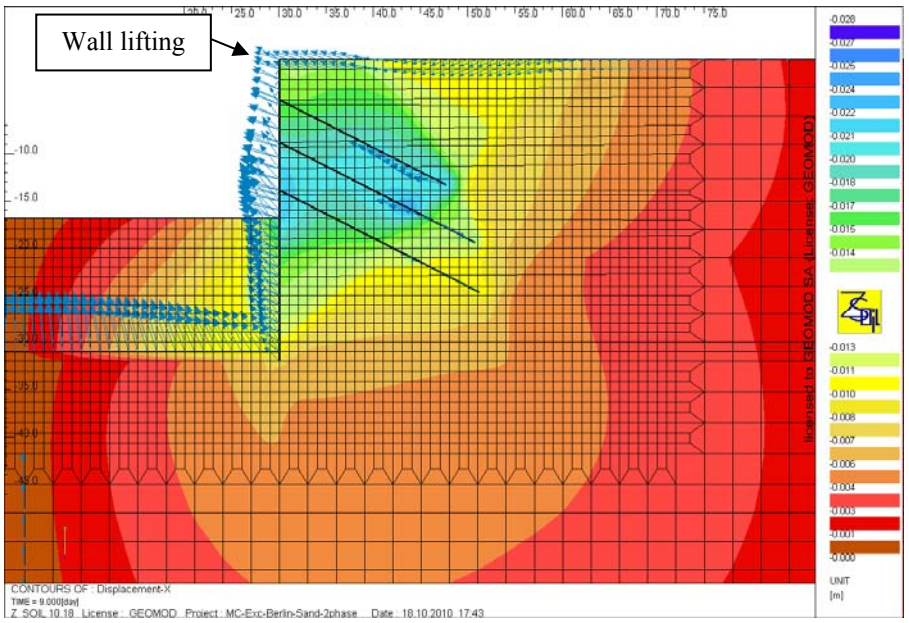
As an enhanced version of the HS-Standard model, HS-SmallStrain accounts for small strain stiffness and therefore, it can be used to some extent to model hysteretic soil behaviour under cyclic loading conditions. A recent extension of the HS model which has been implemented in ZSoil (the densification model by Zienkiewicz [10]) includes liquefaction effects.

4. Practical applications of the HS model

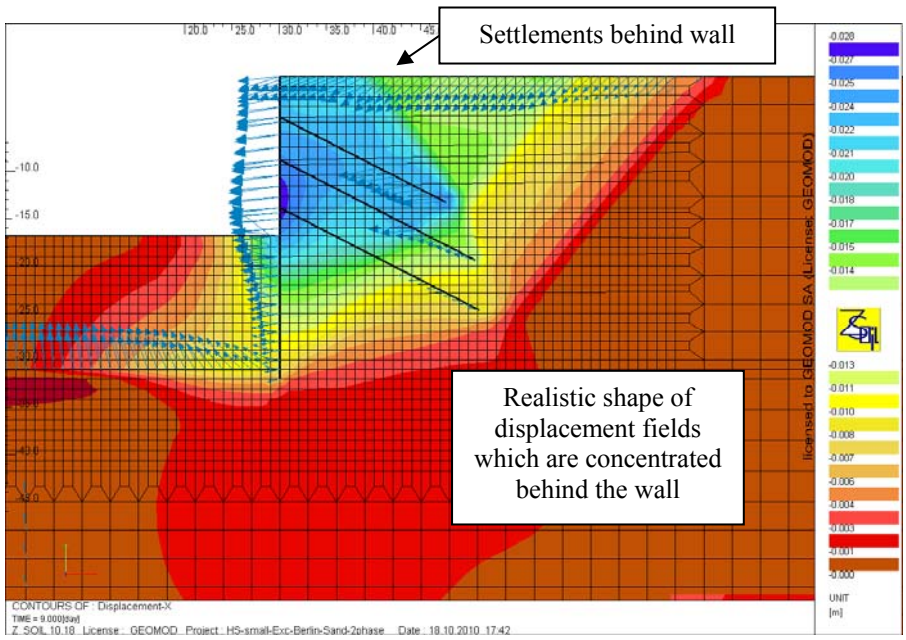
4.1 Retaining wall excavations

The differences in predictions between the HS model and the basic Mohr-Coulomb model can be illustrated on a FE model of a deep excavation in Berlin Sand - a benchmark problem distributed by [8]. Details of FE modelling for this example can be found in the ZSoil technical report [7].

Figure 9a demonstrates the results of computing with the Mohr-Coulomb model and the effect of wall lifting which has been discussed in Section 3.2. On the other hand, realistic settlements behind the retaining wall, as well as, expected directions of wall displacements are obtained with the aid of the Hardening Soil model.



a)



b)

Figure 9 Comparison of numerical predictions of horizontal displacements for the excavation in Berlin Sand: (a) Mohr-Coulomb, (b) Hardening Soil SmallStrain.

Considering mathematical complexity of the Hardening-Soil model, one may expect an increased computational effort. Indeed, the user may observe an increased number of iterations for each computational time increment comparing to modelling using simple models, but in times of fast personal computers this factor does not play an important role.
The example of a complex 3D modelling of 15-20-meter deep excavation is presented in Figure 10. Despite a large number of continuum elements and structural elements like shells and trusses, computing of a phase-by-phase excavation took about 5 hours on a 32-bit system with the aid of 4-core processor and 4Gb of RAM memory.

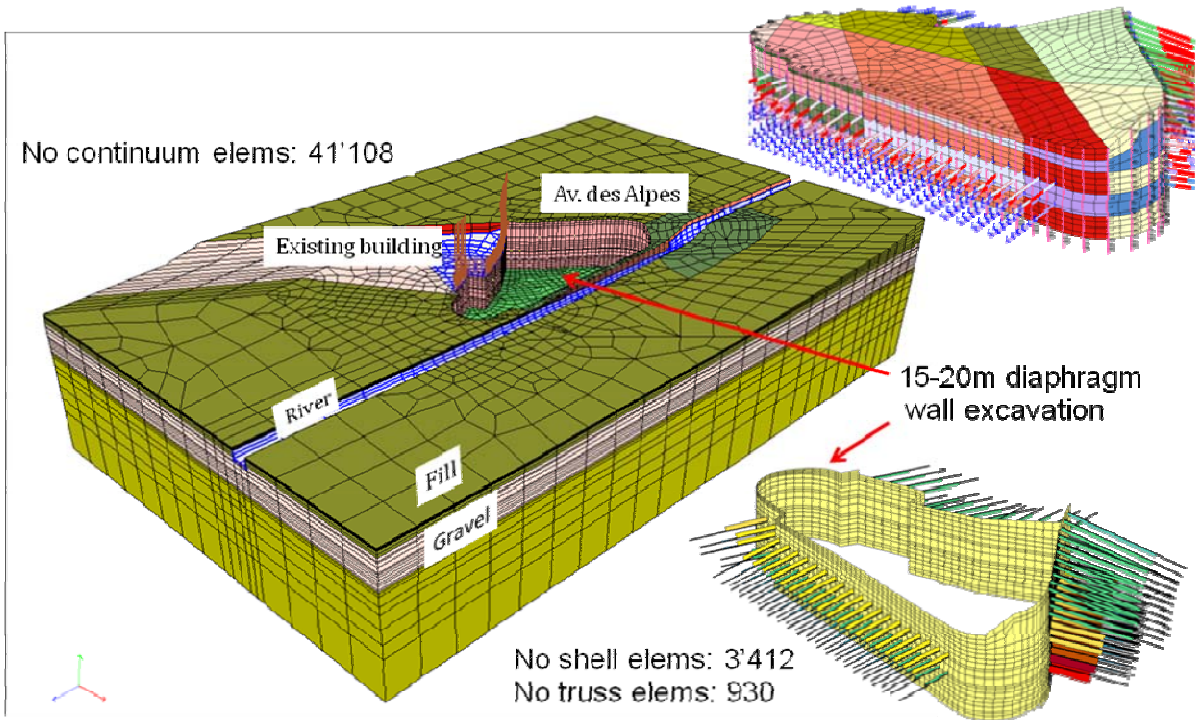


Figure 10 3D modelling of retaining wall excavation in Montreux (GeoMod SA archives).

Figure 11shows a 2D section for the FE model presented in Figure 10. It can be again observed that the use of the HS model allows reliable modelling of the settlements behind the excavation wall, as well as the domination of horizontal displacements for the diaphragm wall without the effect of wall lifting.

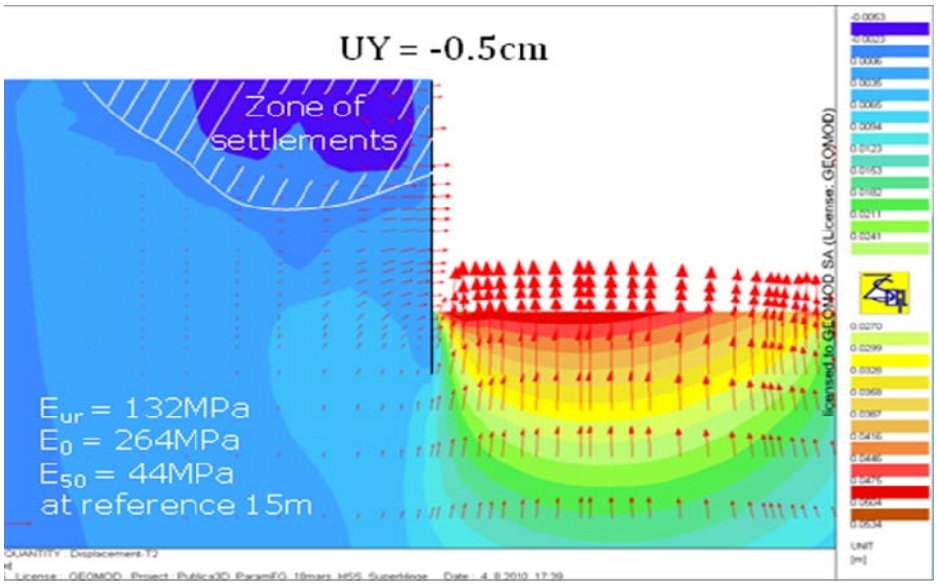


Figure 11 2D section with fields of vertical displacements and displacement vectors for the FE model presented in Figure 10.

4.2 Tunnel excavations

Tunnel excavations in densely built-up urban areas require a careful and precise deformation analysis taking into account the presence of existing buildings which are located next to the excavated zone. The underestimation of settlements for surrounding subsoil may thus have an important influence on the state of existing structures. Hence, reliable modelling of tunnel excavation requires advanced analysis which accounts for pre-failure non-linear soil behaviour.

This example demonstrates the importance of modelling tunnel construction problems with the use of advanced constitutive models such as Hardening Soil model. The example highlights the differences in predictions of subsurface displacements during tunnel excavations in the stiff, heavily overconsolidated London Clay modelled with (a) linear-elastic, perfectly plastic Mohr-Coulomb model, and (b) non-linear elastic, perfectly plastic models: HS-Std and HS-SmallStrain.

This study reanalyzes the excavation model of the twin Jubilee Line Extenesion Project tunnels beneath St James's Park (London, UK) which has been reported in the original paper by [2].

The problem statement, i.e. subsurface stratigraphy and the orientation of tunnels is presented in Figure 12 and Figure 13. Numerical modelling involved coupled hydro-mechanical analysis (consolidation). Stiffness parameters for the HS model were calibrated using laboratory ε_1-q data points for the isotropically consolidated undrained extension triaxial test (CIEU), Figure 14 and Figure 15. It can be noticed that the model well reproduces strong stiffness variation which were observed in laboratory tests.

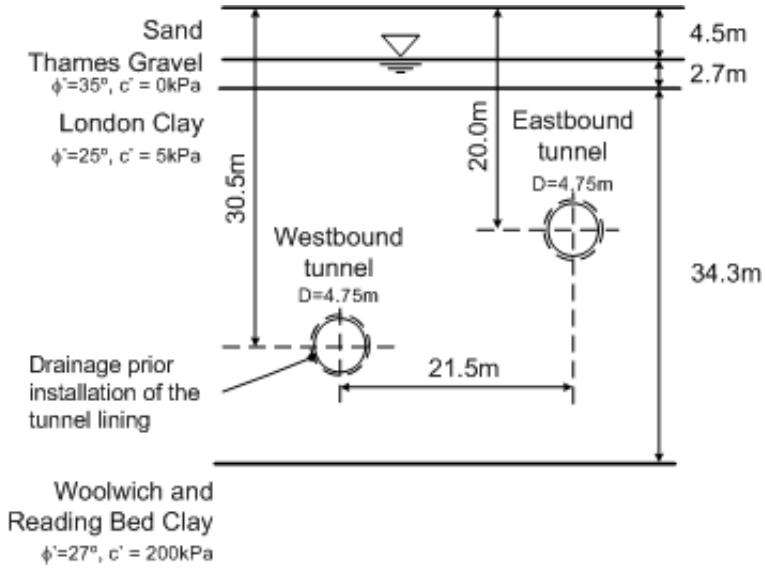


Figure 12 Soil stratigraphy and diagonally oriented tunnels at St James's Park, London, UK.

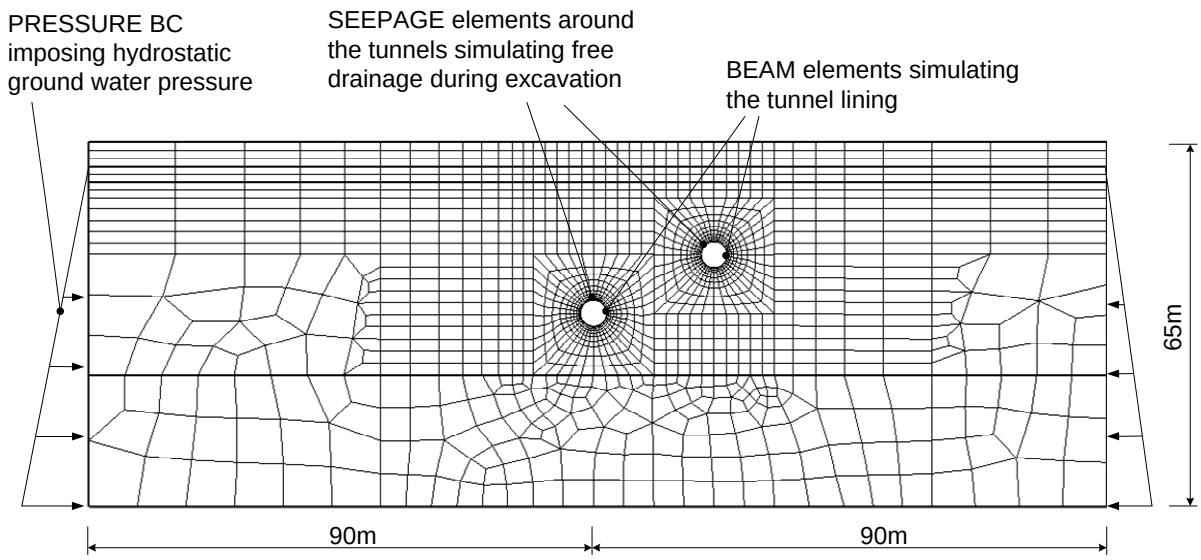


Figure 13 FE mesh for tunnel excavation in London Clay.

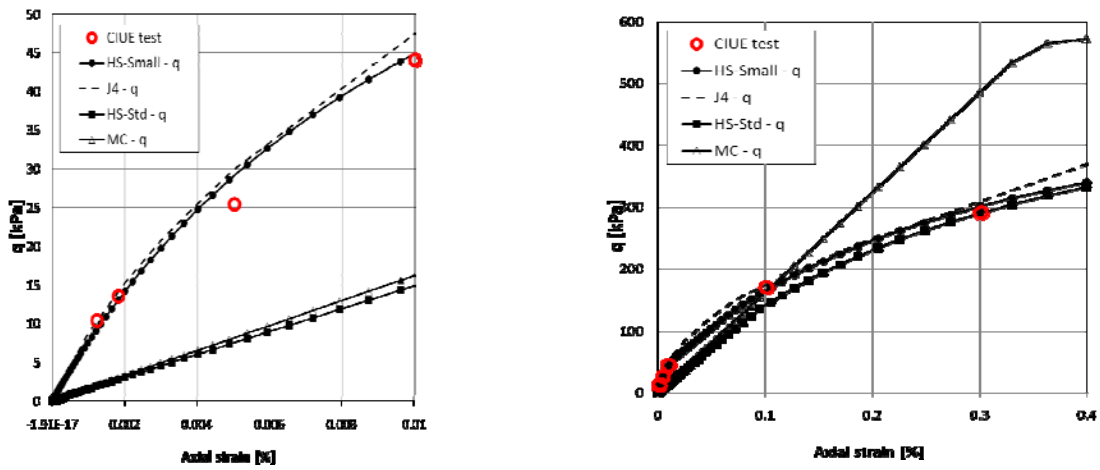


Figure 14 Stress-strain curves: comparison between non-linear models (HS-Std, HSSmallStrain,and J-4 model), linear Mohr-Coulomb model and laboratory test data points obtained in the isotropically consolidated undrained extension test.

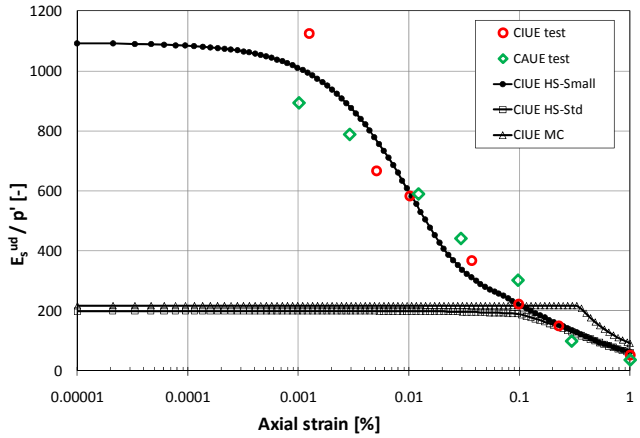


Figure 15 Variation of the undrained secant stiffness-strain curve $\varepsilon_1 - E_s^{ud}$: comparison between numerical models and laboratory data points obtained in the isotropically (CIEU) and anisotropically (CAEU) consolidated undrained extension tests.

Figure 16 presents numerical predictions for the surface settlement profiles after excavation of the 1st (westbound) tunnel. It can be noticed that predictions generated by the M-C model are strongly underestimated compared to the field data. Non-linear pre-failure analyses predict deeper and narrower profiles. HS-SmallStrain model gives narrower shape of surface settlements than HS-Std. The initial higher stiffness of the HS-SmallStrain concentrates the strain levels at the unloading boundary giving slightly deeper profile than HS-Std, and therefore the displacements from 10m-offset from the tunnel axis are reduced further away to the mesh sides.

Figure 17 shows that in the case of the Mohr-Coulomb model, soil displacements around the excavated tunnel can be significantly smaller than those predicted by the HS-SmallStrain model. Moreover, decomposition of absolute displacements along horizontal and vertical directions which is presented in Figure 18, shows considerable differences in soil movements in both directions.

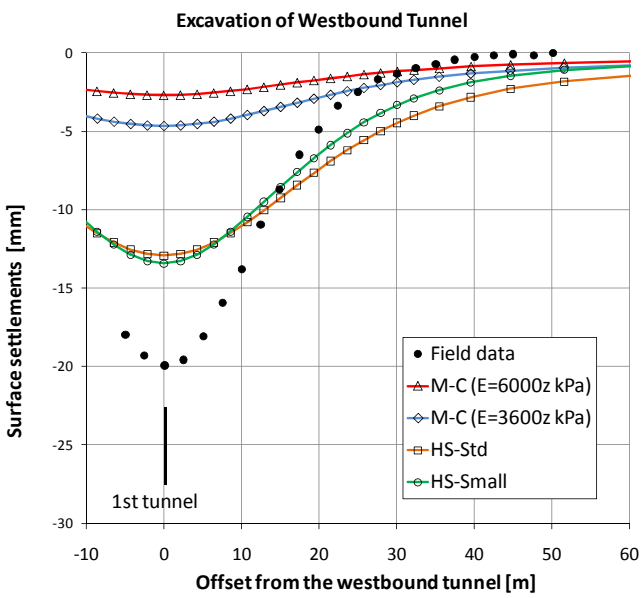


Figure 16 Surface settlement profiles after excavation of 1st tunnel: comparison for different models.

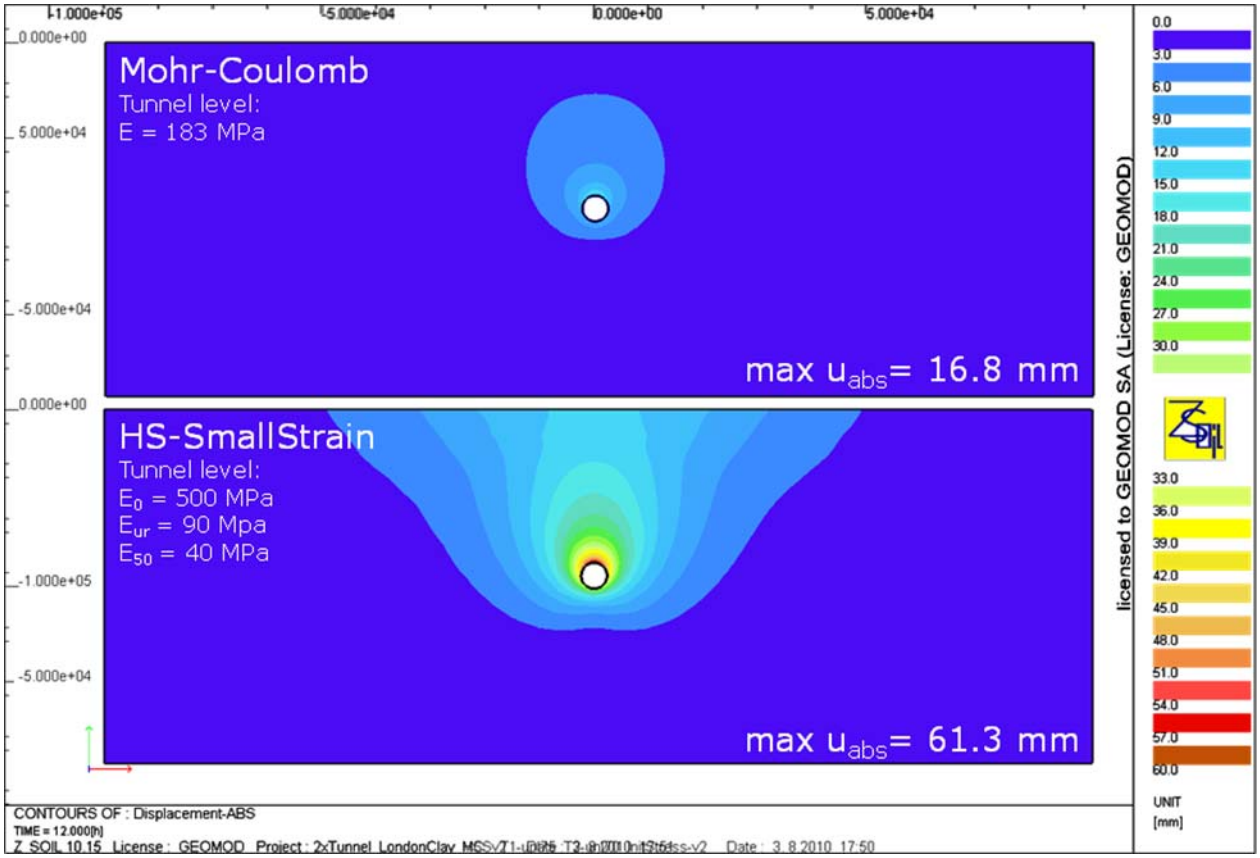


Figure 17 Comparison of the absolute displacement fileds around the excavated westbound tunnel.

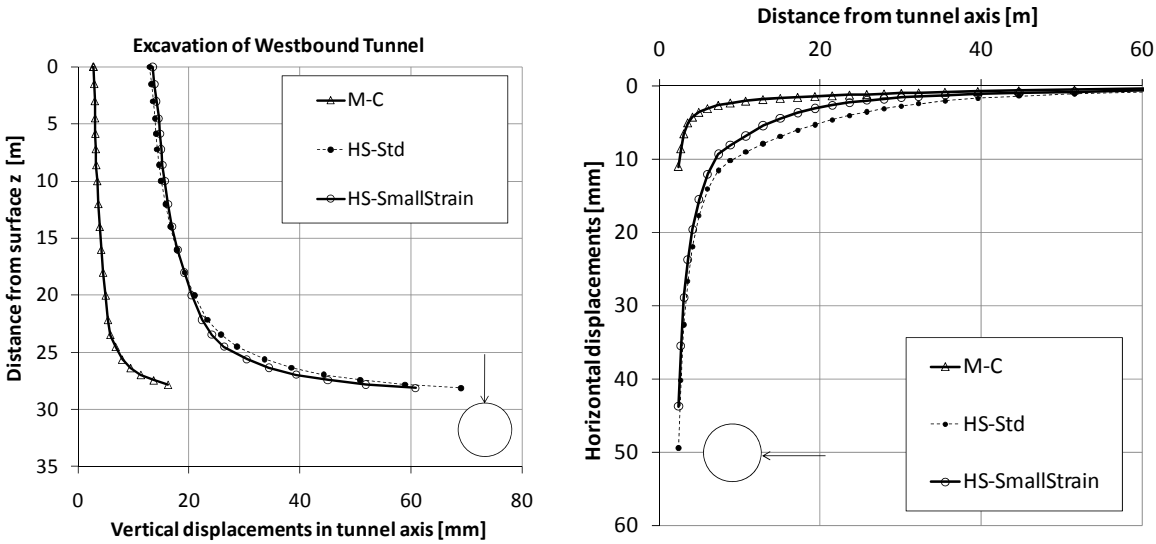


Figure 18 Excavation of the westbound tunnel: (left) vertical settlement in the tunnel axis; (right) horizontal displacements along tunnel axis level of the westbound tunnel.

4.3 Undrained loading and consolidation

The application of the HS-model for a consolidation problem is presented on a trial embankment problem near Almere, Netherlands. The analysis compares results derived from two simulations with the Mohr-Coulomb model and the Hardening-Soil SmallStrain model.

The topology of the embankment problem is illustrated on Figure 19. The upper layer consists of a ligthly-overconsolidated organic clay layer (OCR=2) which is deposited on a stiff sand layer. The embankment is constructed up to a height of 2 m. The material involved is taken from the upper soil layer. After the construction, the embankment is backfilled and the clay layer follows consolidation.

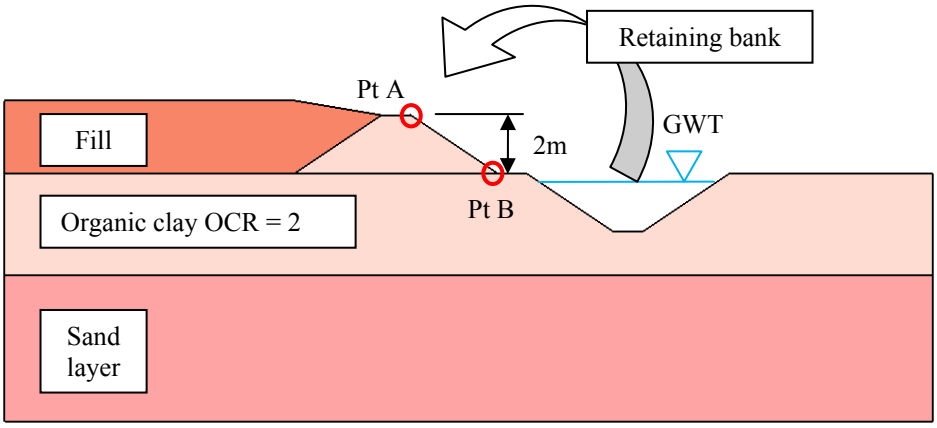


Figure 19 Topology of an embankment problem at Almere, the Netherdlands.

The stiffness characteristics for were taken as follows:

- for the M-C model: constant Young modulus E^{MC}=13.75 MPa.
- for the H-S model the stiffness moduli were specified for the reference minor stress σ_{ref} = 30kPa: the unloading-reloding modulus twice as Young modulus for M-C ($E_{ur} = 2E^{MC}$) , the secant modulus $E_{50} = E^{MC}/2$, the initial stiffness $E_0 = 2E_{ur}$, and tangent oedometric modulus $E_{oed} = E_{50}$.

Figure 20 presents numerical predictions of displacements for the crest and base of embankment. The initial branches of curves are related to the back-filling process which corresponds to the *undrained loading*. It can be noticed that for both measuring points, HS model generates superior displacements since the undrained loading is primarily deviatoric and the shear hardening mechanism of the model is activated. Activation of this mechanism mobilizes degradation of soil stiffness and the non-linear relation ε_1-q can be reproduced. Clearly, this relation for the M-C model is linear.

The second part of the curves is related to *consolidation*, i.e. increasing effective mean stress *p'* and deviatoric stress *q*. For this part, both models show essentially the same response in terms of vertical and horizontal displacements increments since the material is overconsolidated (OCR = 2 for HS model) and consolidation does not involve volumetric plastic straining. On the other, by changing the degree of preconsolidation to almost normally-consolidated state (OCR=1.2), consolidation activates volumetric hardening plasticity and a considerable increase of settlements can be observed.

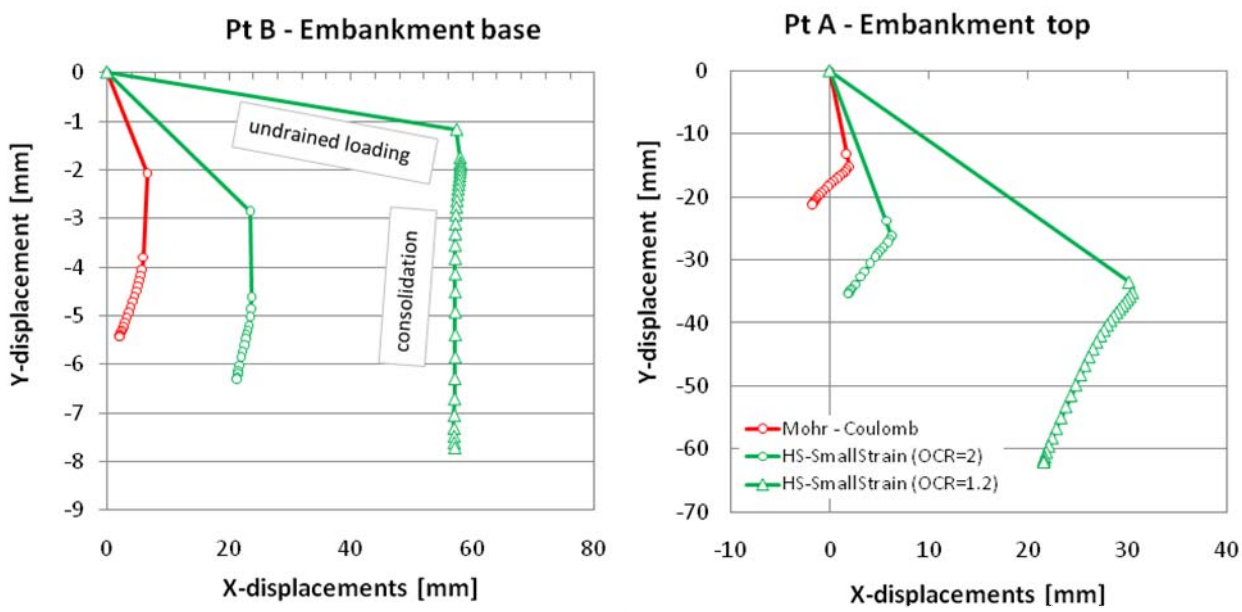


Figure 20 Comparison of numerical predictions for displacements at the top and the base of an embankment using Mohr-Coulomb model and HS model with OCR=1.2 and OCR=2.0.

5. Summary

This article highlights an importance of using the advanced Hardening Soil model in numerical modelling of typical geotechnical problems. It has been illustrated that the model:

- correctly reproduces the strong reduction of soil stiffness with increasing shear strain amplitudes;
- can be recommended for Serviceability Limit State analyses as predicted soil behavior is more closely matched to field measurements than basic linear-elasticity models;
- is applicable to most soils as it accounts for pre-failure nonlinearities for both sand and clay type materials regardless of overconsolidation state.

Despite the mathematical complexity of the HS model, its parameters have explicit physical meaning and can be determined with conventional soil tests or they can be estimated following geotechnical evidence (cf. [3]).

6. References

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